Toronto, Canada
International Symposium on Room Acoustics
2013 June 9-11

Design-focused acoustic analysis of curved geometries using a differential raytracing technique

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ABSTRACT

Curved surfaces have a major influence on the propagation of sound in rooms. Convex surfaces always lead to attenuated reflections spread out over a larger area, while concave surfaces create either amplification or attenuation of reflections, depending on the source and receiver positions relative to the curved surface. Contemporary architecture often involves extensive use of both convex and concave surfaces. Therefore, acoustic consultants need to be able to quickly gain a thorough understanding of a given curved geometry, and judge to what extent the arising focusing will be either benign or problematic (due to focusing, inhomogeneous sound distribution or flutter echoes).

The advent of NURBS-based 3D software (e.g. Rhino [5]) has made it possible to carry out very precise geometrical raytracing on curved surfaces, revealing their geometrical coverage. As a further refinement, a differential raytracing technique is proposed, allowing the straightforward calculation of the amplification or attenuation created by a given curved surface. This purely geometrical technique can also be applied to higher order reflections, in order to help identify flutter echo problems created by multiple reflection paths involving concave surfaces.

Practical examples of halls with significant curved geometries will be given to illustrate the use of this approach, including Wigmore Hall, London and Wuxi Grand Theatre.

1 INTRODUCTION

Anyone with a basic understanding of acoustics (or optics) knows that curved surfaces create convergence or divergence of sound (or light) energy. However, very few acousticians tend to quantify the degree of such convergence or divergence. This can probably be partly explained by the discrete nature of the underlying techniques used in the software packages that are used in the day-to-day practice of acoustic consultancy.

Advanced raytracing based software applications, such as Odeon and CATT, are great tools to accurately simulate impulse responses between given source and receiver pairs and derive corresponding room acoustic parameters. However, since only flat surfaces can currently be handled by the algorithms of such software, a single curved surface needs to be facetted into many flat surfaces. This allows capturing the overall effects of focusing in the impulse responses as a good approximation, but it doesn't give a clear and simple quantification of amplification created by a certain curved surface.

Numerical methods solving the wave equation, such as Finite Elements (FEM), Boundary Elements (BEM) or Finite Differences (FD), allow precise calculation of the acoustic behaviour of any given shape with any given surface impedance, taking into account wave phenomena (diffraction, scattering and interference effects). Any curved geometry can be handled provided the discretisation step of the element is sufficiently fine to represent the curved geometry and is adapted to the frequency. The question arises to what extent such simulations should become the de facto calculation tool for acousticians. It would be interesting to find out how many acousticians are currently already using these powerful tools for time efficient optimisations of architectural designs.

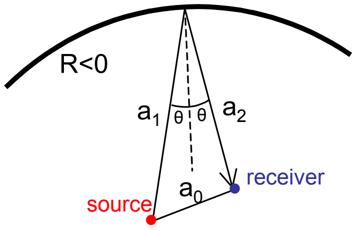
As will be shown in the following, geometrical acoustics, despite being a high-frequency only approximation, can also be used to enlighten the behaviour of curved surfaces - both qualitatively and quantitatively - especially when implemented in NURBS-software. The rest of this article will only deal with purely geometrical reflections. It should of course be borne in mind at all times that this represents an approximation, and that the real acoustic behaviour – including diffraction and interference effects - is more complex.

2 ESSENTIAL BASICS OF CURVED SURFACES

Convex surfaces always create spreading-out and hence attenuation of the sound energy density associated with the reflection in comparison with a flat surface.

Concave surfaces on the other hand are more complex: they always create convergence and hence amplification of the reflected sound energy density in certain areas, but also attenuation in other areas. Concave surfaces typically have foci associated with the shape itself (e.g. circle or ellipse). The reflective behaviour, however, does not solely depend on the shape, but also on the position of the source relative to the curved surface. In other words, the acoustic foci will be in different locations than the foci associated with the shape and they depend on the location of the source [3]. Different reflection types, corresponding to different cone sections, can be distinguished by locally approximating a (part of a) concave surface by a circle with radius R, and by determining the source location relative to the centre point of this circle [1]. This qualitative approach can yield a better understanding of how focusing works, at least for first order reflections.

Rindel [2] published a purely geometrical and highly practical approximation to quantify the strength of reflections off a curved surface (again only for first order reflections). To this end, he attributed a term ΔL_{curv} to the strength of a bouncing ray. A positive ΔL_{curv} represents amplification of sound energy due to the curved surface, whereas a negative ΔL_{curv} denotes attenuation of sound energy due to the curved surface. See Figure 1.



$$\Delta L_{curv} = -10\log\left|1+\frac{a^*}{R\cos\theta}\right|$$

$$a^* = \frac{2a_1a_2}{a_1+a_2}$$

Figure 1: First order reflection off a circular arc, for given source/receiver locations.

Since the advent of NURBS-based 3D design software during the last decade, it has become possible to carry out precise raytracing on curved surfaces. Next, using a principle that we will call “differential raytracing” it is possible to extend Rindel’s approach to any curved shape and to any reflection order, and calculate the term ΔL_{curv} with high precision - albeit of course within the confines of geometrical acoustics.

3 PRINCIPLE OF DIFFERENTIAL RAYTRACING

Consider a reflecting surface and an omnidirectional source S. A sound ray is fired off in the direction of reflection point P and is specularly reflected towards receiver point R, as shown in Figure 2.

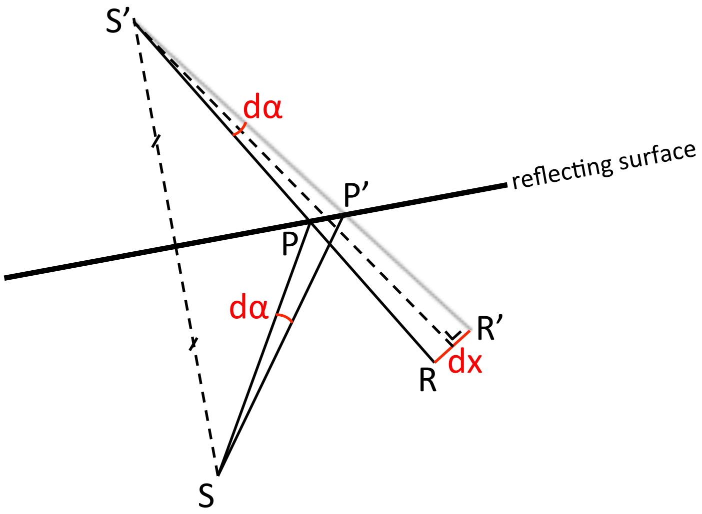


Figure 2: Principle of differential raytracing

Now consider an infinitesimally small angular deviation dα of the direction in which the ray is fired off from the source S. This corresponds to an accompanying “twin” ray, reflected in point P’ and traced until point R’ where the same travelled path distance is obtained as the original ray:

$$|SP|+|PR|=|SP'|+|P'R'| \tag{2}$$

The infinitesimally small angular deviation dα hence creates an infinitesimally small displacement dx of the receiver point:

$$dx = |RR'| \tag{3}$$

The image source of S associated with the reflecting surface is called S'. According to image source theory,

$$\begin{aligned} |SP| &= |S'P| \\ |SP'| &= |S'P'| \end{aligned} \tag{4}$$

When combining (2) and (4), it is clear that the triangle S'RR' is an isosceles triangle, with equal sides S'R and S'R'. Splitting it into two identical right triangles, elementary trigonometry can be applied:

$$dx = 2|S'R|\sin\left(\frac{d\alpha}{2}\right) \tag{5}$$

For a flat reflecting surface, equation (5) gives the displacement dx_{flat} of the receiver point R as a response to a small angular change dα. For a curved surface, it is straightforward to calculate the displacement dx_{curv} of the receiver point as a response to the same angular change by means of NURBS-software, capable of accurately representing curved surfaces.

In point R, the amplification in dB due to the curved surface (i.e. the strength of the reflection off the curved surface relative to the strength of the reflection off a flat surface) can then be evaluated as:

$$\Delta L_{curv} = 10\log\left(\frac{dx_{flat}}{dx_{curv}}\right) \tag{6}$$

It can be shown that the reasoning as shown above for 1^{st} order reflections in 2D holds true for any reflection order. In addition, a similar principle can be derived in 3D by considering infinitesimal angle changes dφ and dϑ of the two angles φ and ϑ in a spherical coordinate system, and tracking the corresponding surface area changes of an infinitesimally small projected triangle.

A 2D example of NURBS based raytracing (both with and without the differential method) is presented in Figures 3a and 3b for a hypothetic plan with curved surfaces. The source is indicated by the black dot in the left half of the room and only first order reflections are considered. For didactic purposes the two flat surfaces have been deliberately made 100% absorbing in order to let the focusing effects stand out.

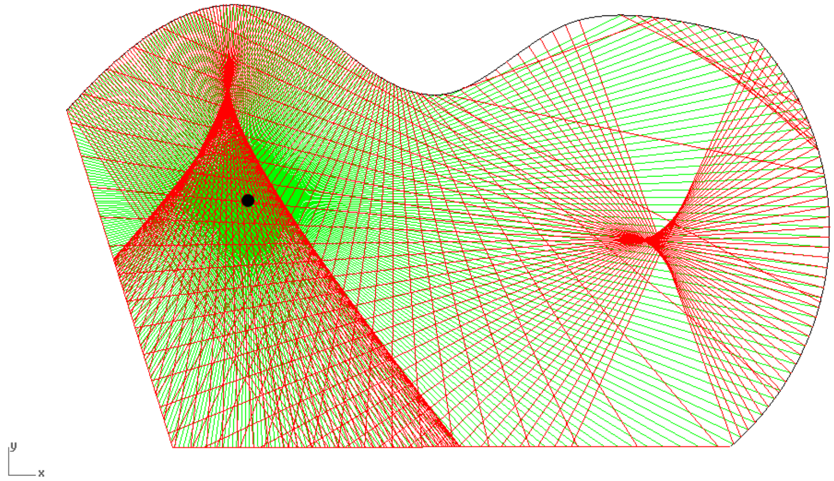


Figure 3a: NURBS based raytracing in a hypothetical plan containing curved surfaces. Only direct sound (green) and first order reflections (red) are shown. It can clearly be seen where focusing occurs, i.e. in the zones where red rays converge.

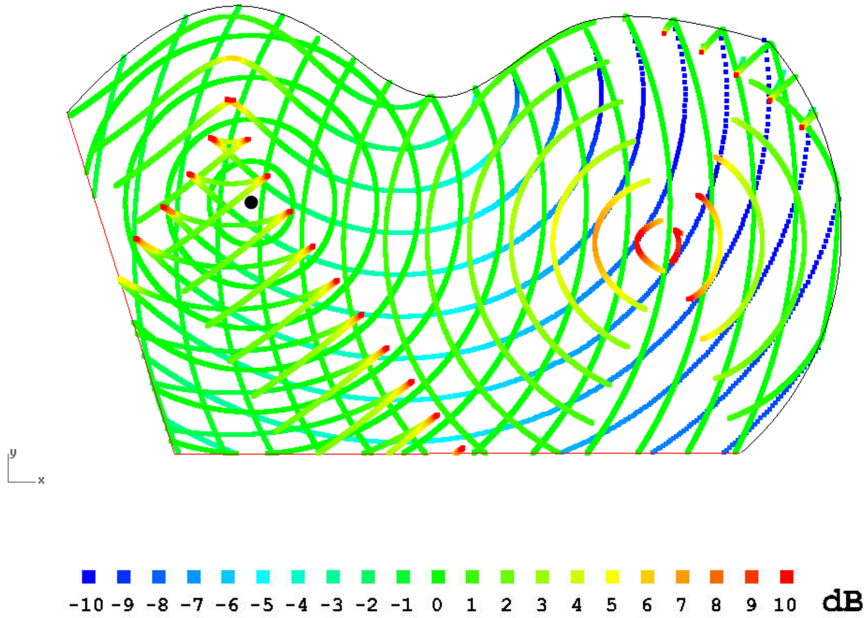


Figure 3b: Plot of the term ΔL_{curv} (in dB) calculated using differential raytracing. The 0dB reference corresponds to a reflection off a flat surface. Again, only up to first order reflections are considered in this example. The red colour clearly shows where convergence of sound energy occurs due to the curvature, the blue colour where divergence occurs.

NB. ΔL_{curv} is of course not directly linked to what is heard in those locations. In the analysis of a focusing problem it represents only one of several elements in a more complex whole. For example, the time of arrival and the level relative to the direct sound, and the masking effects of the reverberant field are additional aspects to be considered.

4 A FEW PROJECT EXAMPLES

4.1 Wigmore Hall

The Wigmore Hall in London is a chamber music hall known for its excellent acoustics, despite the concave ceiling and stage apse. Previous work [3] had already shown that the ceiling creates significant focusing, both in the parterre and in the balcony. Applying the differential raytracing technique described in this article to a 3D NURBS-model of Wigmore Hall confirms that for a stage source on the centre line the very top part of the ceiling is responsible for the strongest focusing effects in the parterre (near the centre line), while the rest of the ceiling creates a milder amplification of sound (ΔL_{curv} between 3 - 5dB in the middle audience block). On the other hand, in the rear balcony the strongest focusing originates from the lateral parts of the ceiling (at approximately midway between the top and sides of the ceiling), which is likely to significantly increase loudness and apparent source width in the central parts of the balcony.

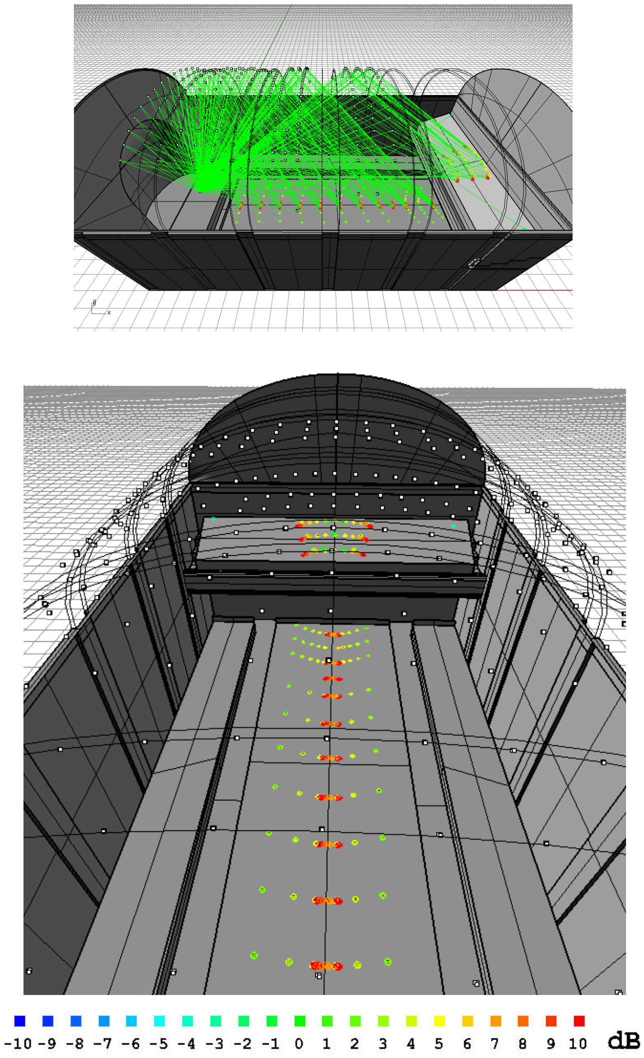


Figure 4: ΔL_{curv} calculated using differential raytracing in a 3D NURBS model of Wigmore Hall. The coloured dots correspond to receiver locations at audience head height.

4.2 Nordlyskatedralen

The design of the new Cathedral of the Northern Lights in northern Norway, having a complex concave-only geometry, was optimised to avoid strong focusing zones and flutter echoes. This was done by a combination of optimising the shape of the church room, using differential raytracing, and applying diffusing elements on the walls. The church opened in February 2013 without any audible trace of focusing or flutter.

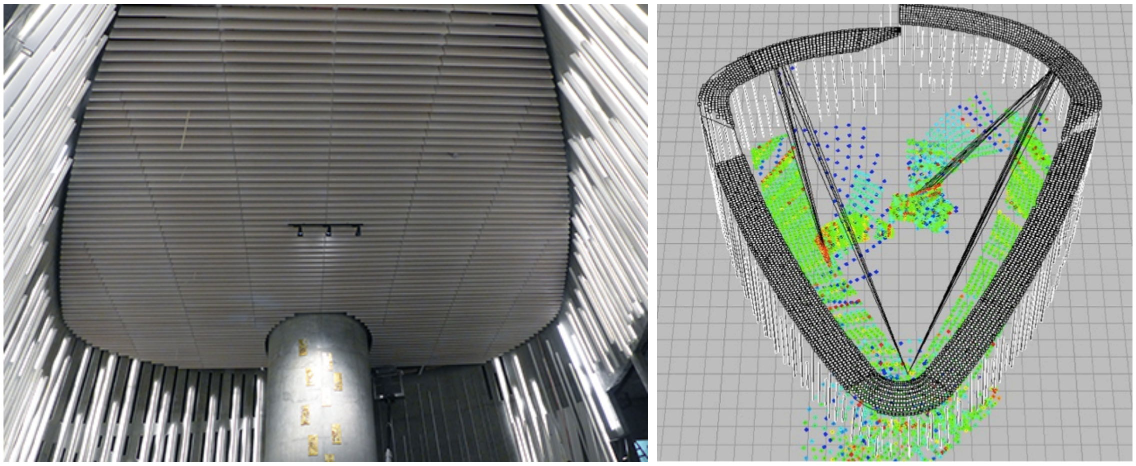


Figure 5: Left: photo of the concave church room. Right: ΔL_{curv} calculations on an intermediate model for second order reflections off a hidden horizontal shelf and the curved walls. The back parts of the shelf were eventually removed from the design when it became clear from the modeling exercise that undesired focusing was likely to occur.

4.3 Wuxi Grand Theatre

The new opera house in Wuxi, China, was designed according to a novel architectural paradigm of stacked curves (the “banded” auditorium) [4]. This presented an opportunity to work closely with the architect (PESark) in optimising early lateral reflections to the audience, created mainly by 2nd order reflections involving the soffits of ledges created by stacking the different curves. Rhino was used to identify which parts of this complex side wall geometry were acoustically efficient, and which parts needed further optimization. The process is comparable to using a laser in a physical scale model.

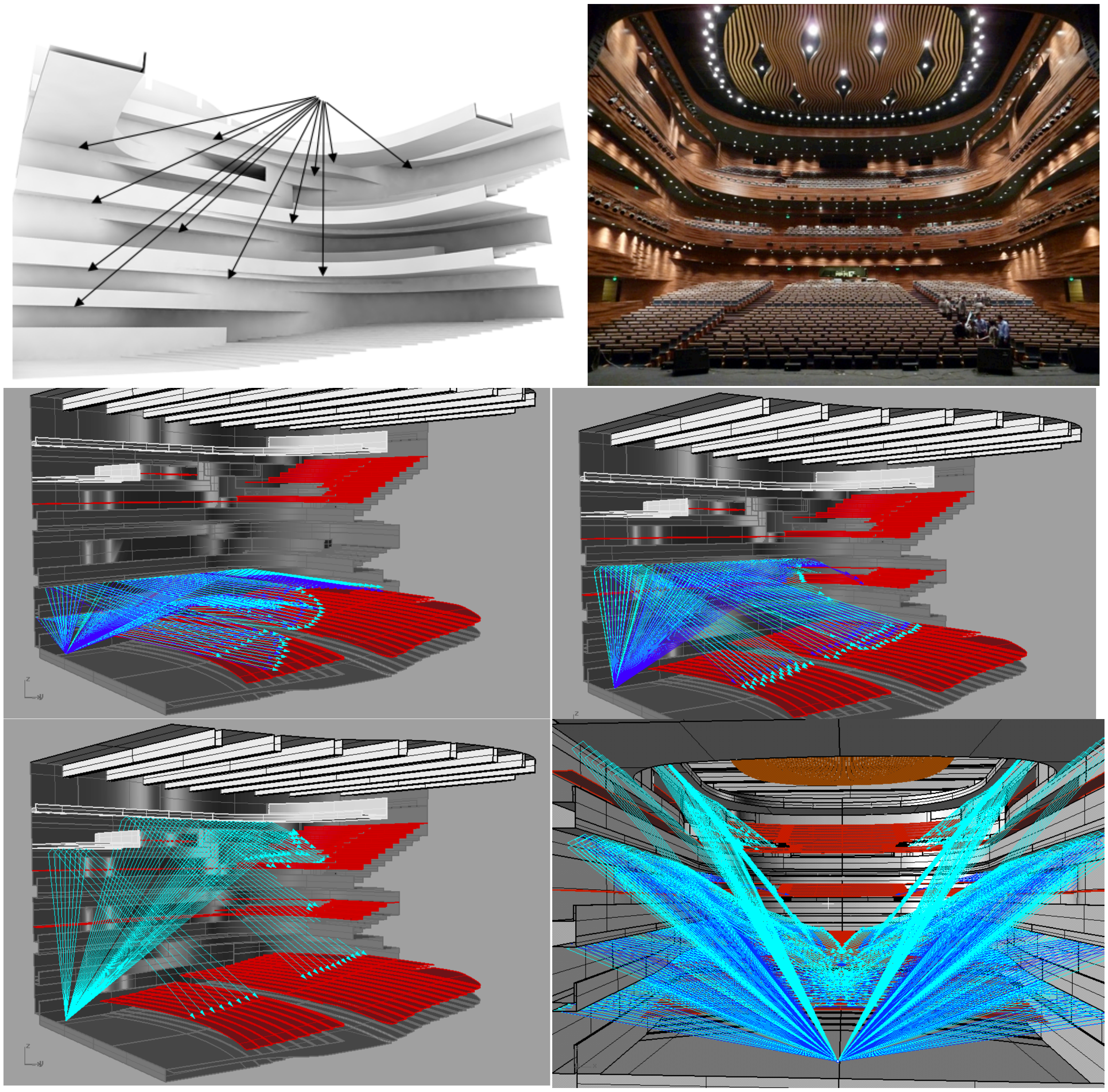


Figure 6: Integrated acoustic and architectural design of Wuxi Opera House, optimized acoustically in Rhino.

ACKNOWLEDGMENTS

Dr. Raf Orlowski is credited for having triggered and supported the first author's interest in the subject of curved surfaces during their time together at Arup Acoustics. In addition all authors would like to thank Olivier Terny for his valuable help on the Rhinoscript coding.

REFERENCES

1 L. Cremer & H. Müller, 'Principles and Applications of Room Acoustics, Volume 1', Applied Science Publishers Ltd (English edition 1982), chapter I.3.2

2 J.H. Rindel, "Attenuation of Sound Reflections from Curved Surfaces", Proceedings of 24th Conference on Acoustics, Strbské Pleso, pp. 194-197 (1985)

3 T. Wulfrank, R.J. Orlowski, "Acoustic Analysis of Wigmore Hall, London, in the Context of the 2004 Refurbishment", Proceedings of the Institute of Acoustics Conference (2006), Copenhagen, Denmark

4 E. Kahle, T. Wulfrank, Y. Jurkiewicz, B. Katz, and H. Möller, "Acoustic design of Grand Theatre projects in China," Meeting of Acoustical Society of America (Acoustics 2012), Hong Kong, China.

5 Rhinoceros software: www.rhino3D.com, Robert McNeel & Associates