Architectural shape and early acoustic efficiency in concert halls (L)

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Supplying sufficient early reflections to audience members is an important prerequisite to good acoustic quality in performing arts spaces. However, the relationship between the geometry of a room and its acoustic efficiency in terms of early energy has rarely been investigated using basic geometrical principles. The present study demonstrates the possibility of predicting the average value of early reflected energy across the audience area using solid angles. The formulas obtained display the influence of various factors on average early energy; in particular, the direction of arrival of early reflections is found to play a significant role, which highlights interesting implications for the acoustic design of concert halls. © *2012 Acoustical Society of America*.
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I. INTRODUCTION

Research studies from the second half of the 20th century[1–8] as well as a more recent ones[9,10] have demonstrated that the acoustic success of a concert hall strongly depends on the appropriate design for early reflections, possibly even more than on an appropriate reverberation time. Acoustic consultants have widely accepted this idea and tackle the challenge of developing room geometries that will supply the appropriate early reflections. In very large concert halls seating more than 2000, providing a sufficient amount of early energy to audience members is one of the prime challenges.[11,12] But in some cases, the question arises whether providing an excessive amount of early reflections might alter the late response of the hall.

During the development of a concert hall design, verifying that the geometry delivers the appropriate amount of early energy requires either 3D simulations using a ray tracing software or a scale model measurement campaign. Both options are time consuming, and these tools can never be used to systematically test all design options: architects and clients need quick answers when taking critical decisions for the design of a hall. The purpose of this publication is to highlight the possibility of developing a new tool that may bring an interesting change in the acoustic design methods.

A previous study[13] demonstrated a good correlation between the total surface area of efficient reflectors in a hall and the mean value of measured clarity C80. The correlation was further improved when using a simplified definition of the solid angle of efficient reflectors. The previous study was purely empirical, based on the observation that the average amount of early energy across the audience should be linked somehow to the percentage of energy emitted by the source that is reoriented towards the audience by efficient reflective surfaces. In the current study, a more complete relationship between the amount of early energy received by the audience in a hall and a geometrical parameter based on solid angles will be derived mathematically under the assumptions of geometrical acoustics.

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II. A RELATIONSHIP BETWEEN EARLY ENERGY AND SOLID ANGLES

In a room, several reflective surfaces are oriented in a way so as to provide early reflections from a point source on stage to some parts of the audience. These reflective surfaces will be named "efficient surfaces." The amount of energy emitted by the non-directional sound source and received by a given reflective surface S_i is proportional to the solid angle Ω_i of this surface measured from the source point. The value of Ω_i can be obtained using $\Omega_i = S_i \cos(\theta r_i)/R_i^2$, as long as S_i is significantly inferior to the square of its distance to the source R_i (θr_i being the angle of incidence of the wavefront on S_i). The total efficient solid angle Ω_{eff} can then be defined as the solid angle of all efficient surfaces measured from a sound source.

Before deriving the formula relating the average early acoustic energy received by the audience to the efficient solid angle, a question needs to be investigated: what is audience from an acoustic perspective? Areas occupied by audience in a concert hall are generally modeled as planes with absorptive properties. When a planar wave characterized by an intensity vector $\vec{\text{I}}$ hits a small fraction of an audience plane characterized by a vector $\text{d}\vec{\text{S}}a$ (the norm of $\text{d}\vec{\text{S}}a$ being the area of the surface element, and its direction being normal to the surface element, see Fig. 1), the amount of acoustic energy received per second by this surface element is $|\vec{\text{I}}\text{d}\vec{\text{S}}a| = I\text{d}Sa\cos(\theta a)$[14] ($\theta a$ being the angle of incidence of the wavefront on Sa). But audience members are not just surface elements. Human ears—as well as omnidirectional measurement microphones—are pressure-sensitive. The loudness of a planar wave coming to our ears relates to the norm of the intensity vector, not to its scalar product with a normal vector. One audience member included within an audience plane and submitted to an incident planar sound wave can be considered as a small surface receiving an amount of energy that depends on the angle of incidence of the sound wave; but—provided

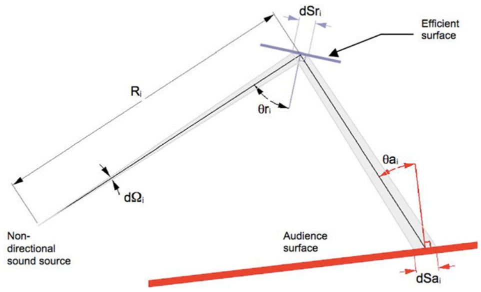

FIG. 1. (Color online) Illustration of the variables used for the geometrical demonstration.

that grazing incidence attenuation can be neglected—the intensity of sound at his ears is independent of that angle of incidence. This observation is the basis for the mathematical demonstration that follows, and seems to not have been taken into account in most literature on room acoustics design.

As a criterion for early energy it is proposed to use the mean value of early G, $G_{\rm em}$. This parameter is derived from the classical formulation of the strength parameter G and quantifies the mean value of early reflected sound strength across the audience.

$$G_{\rm em} = 10 \times \log(I_{\rm em}/I_{10}), \tag{1}$$

where $I_{\rm em}$ is the average over the audience areas of the total reflected acoustic intensity reaching the audience ears before a specific delay D after the arrival of direct sound (and not including the direct sound):

$$I_{\rm em}=\frac{\displaystyle\iint_{\vec{\rm r}\in\,\text{audience}}\left(\int_{t=0+}^{D}\frac{p(t,\vec{\rm r})^2}{\rho_0 c_0}\,{\rm d}t\right){\rm d}S(\vec{\rm r})}{\displaystyle\iint_{\vec{\rm r}\in\,\text{audience}}{\rm d}S(\vec{\rm r})}.$$

And I_{10} is the acoustic intensity created by the same non-directional sound source in free field at a distance of 10 m.

D is the limit between early and late sound, which can be set to 50 ms, 80 ms or any other value which is found appropriate depending on the context. D should be chosen small enough for the diffuse sound field to be neglected. Direct sound is intentionally not taken into account, as it is obviously not impacted by the design of reflective surfaces.

The efficient surfaces can be divided into infinitesimal surface elements dSr. Under the assumptions of geometrical acoustics, each of these surface elements dSr_i directs an acoustic intensity I_i to a corresponding infinitesimal part of the audience dSa_i (see Fig. 1). The average early acoustic intensity across the audience can then be expressed:

$$I_{\rm em} = \sum I_i {\rm d}Sa_i/S_{\rm aud}. \tag{2}$$

If absorption and scattering effects are neglected, the total acoustic energy reaching the audience due to a specific effective surface element dSr_i is the same that had reached this surface element, coming directly from the source. Neglecting all scattering effects requires that the size of all reflecting surfaces is large compared to the wavelength. As all models based on geometrical acoustics, the validity of this development is consequently limited to high frequencies.

The conservation of energy within this acoustic beam can be obtained from: $|\vec{\rm I}_{\rm i}{\rm d}\vec{S}a_i| = |\vec{\rm I}(R_i){\rm d}\vec{\rm S}r_i|$, where $I(R_i)$ is the intensity created by the sound source on dSr_i, and where surface elements are vectors, oriented by their surface normal.

The following formula is then obtained for I_i:

$$I_i = I(R_i){\rm d}Sr_i\cos(\theta r_i)/{\rm d}Sa_i\cos(\theta a_i), \tag{3}$$

θr_i and θa_i being the angles of incidence as defined on Fig. 1. The sound source being non-directional, $I(R_i)$ can be expressed as a function of R_i and a constant E_0 related to the sound power of the source. It is then obtained

$$I_i = (E_0/4\pi R_i^2)[{\rm d}Sr_i\cos(\theta r_i)/{\rm d}Sa_i\cos(\theta a_i)]. \tag{4}$$

Using Eqs. (2) and (4), $I_{\rm em}$ can be expressed as follows:

$$I_{\rm em} = \frac{E_0}{4\pi S_{\rm aud}}\sum\frac{{\rm d}Sr_i\cos(\theta r_i)}{R_i^2\cos(\theta a_i)} = \frac{E_0}{4\pi S_{\rm aud}}\sum\frac{{\rm d}\Omega_i}{\cos(\theta a_i)}. \tag{5}$$

One is then tempted to define a corrected efficient solid angle:

$$\Omega_{{\rm eff},c} = \sum\frac{{\rm d}\Omega_i}{\cos(\theta a_i)}. \tag{6}$$

$\Omega_{{\rm eff},c}$ being the total efficient solid angle, corrected by the angle of incidence on the audience planes of the corresponding reflections. This expresses that acoustic energy emitted by the source can have a different impact as a function of angle of incidence on the audience.

From Eqs. (1), (5), and (6), the following formula is obtained for mean early reflected strength:

$$G_{\rm em} = 10\log(100\,\Omega_{{\rm eff},c}/S_{\rm aud}). \tag{7}$$

In order to clearly distinguish between solid angle of reflectors and angle of incidence with respect to receivers, a different formulation is used in the remainder of this paper. It requires the definitions of the angle θ_m:

$$\frac{1}{\cos(\theta_m)} = \sum\frac{{\rm d}\Omega_i}{\cos(\theta a_i)}\Big/\sum{\rm d}\Omega_i = \Omega_{{\rm eff},c}/\Omega_{\rm eff}. \tag{8}$$

The expression for mean early reflected strength is then:

$$G_{\rm em} = 20 + 10\log(\Omega_{\rm eff}) - 10\log\big(\cos(\theta_m)\big) - 10\log(S_{\rm aud}). \tag{9}$$

III. DISCUSSION

It should be emphasized that $G_{\rm em}$ is a monaural parameter. Just as the more traditional acoustic strength parameter (G), $G_{\rm em}$ can be measured using omnidirectional sound source and microphone and predicted in a ray tracing software. The formula that has just been demonstrated was derived only from the assumptions of geometrical acoustics. Predictions of acoustic parameters obtained with this formula are consequently virtually identical to those obtained with a ray tracing algorithm that would not take into account the effects of diffusion, grazing incidence attenuation, and HRTF. As most existing acoustic parameters for room acoustics, the definition of G_{em} implies that the sound source is non-directional. However, in the actual context of a concert the amount of energy received will of course be dependent on the directivity pattern of the instruments. The location of efficient surfaces with respect to sound source directivity patterns is then another factor that acoustic designers have to take into account.

According to formula (9), the average value of early reflected energy in a room depends only on 3 architectural parameters: (1) The total surface area of audience S_{aud}, (2) The total efficient solid angle Ω_{eff}, and (3) The average angle of incidence of the early reflections on the audience θ_m.

Since publications by Beranek,[15,16] it is now recognized that the surface area of audience is a better indicator of its total absorption power than the number of audience members: in a room of a given volume and a given seat-count, a more comfortable seating layout will create shorter reverberation times. Formula (9) indicates that very comfortable seating layouts do not only reduce reverberations times but also early energy and strength in general. Spreading audience over wider areas will require higher values of total efficient solid angle to obtain the same average early strength.

The influence of Ω_{eff} on early strength was to be expected. $\Omega_{\mathrm{eff}}/4\pi$ corresponds to the fraction of energy emitted by the non-directional source which is oriented by the room surfaces towards the audience and the total efficient solid angle is by definition limited to a maximum value of 4π, or even 2π in cases where the sound source can be considered as included in an absorptive plane (for example, the location of an instrument within the orchestra). This means that in large concert halls with high values of S_{aud}, good acoustic design can only increase Ω_{eff} up to a certain limit in order to obtain appropriate values for early strength. The existence of a seat count limit for appropriate acoustics in a concert hall without sound reinforcement has always been obvious to acoustic specialists. The solid angle theory developed here confirms that limit and formulates clearly its existence.

This observation highlights the importance of the last architectural parameter θ_m, the average angle of incidence of the early reflections on the audience. Obtaining sufficient early strength in a very large room requires that the early reflections arrive at the listeners' ears from surfaces low in the room (under "shallow incidence"). This finding is rather surprising and in fact contradictory to accepted wisdom in at least part of the acoustic community.

A simple example is illustrated in Fig. 2: an audience plane and two efficient reflectors with different location and orientation. The first reflector provides first order reflections from the zenith ($\theta a = 0°$) whereas the second reflector provides reflections with an angle θa of 60° with respect to the direction normal to the audience plane.

Both reflectors subtend the same solid angle at the source point and, therefore, receive the same amount of acoustic energy from the direct sound. Intuitive reasoning might lead one to think that the reflections from the second reflector are weaker than the ones from the first reflector, as the same amount of energy is more widely spread. This is actually wrong: both surfaces are flat and in the chosen example the sound travels exactly the same distance from the source to the audience through both reflection paths. Both reflectors create reflections of exactly the same strength, but the second reflector provides these reflections to a larger proportion of the audience, which gives it a stronger influence on the average value of sound strength across the audience.

A recent study[17] demonstrated that—due to the shape of the human head—reflections from the side are amplified more than median plane reflections. This effect is of course not taken into account in formula (9). The solid angle approach does not demonstrate an amplification effect of shallow incidence reflections. But in the case of lateral reflections coming from the lower part of the room, the amplification effect of lateral reflections and the spreading effect of shallow incidence will combine positively.

This has very interesting implications for the acoustic design of performing arts spaces: the same amount of acoustic energy generated by a sound source can be used and channeled in order to generate stronger loudness and a better source presence by favoring the creation of reflections coming to the audience from the lower part of the room.

The only valid contradiction to this finding lies in the famous seat-dip effect. It is now well known that sound waves propagating above an audience area under grazing incidence are attenuated. This attenuation is mostly effective at low frequencies but also to a lesser extent at mid-frequencies.[18–20]

This effect is of course not taken into account in the theory developed here as the attenuation of sound under grazing incidence can only be explained by wave theory. The formulas developed on the subject by Cremer[21] can already shed some light on the situation. If the audience is modeled as a totally absorptive plane, attenuation due to grazing incidence tends to infinity at all frequencies when the angle of incidence tends to 90°. This would suggest that, in some cases, grazing incidence attenuation could compensate for the increased influence of grazing incidence reflections on average early reflected strength.

On the other hand, measured data from previously published research[19] indicates that grazing incidence attenuation at mid-frequencies can be neglected for angles of incidence

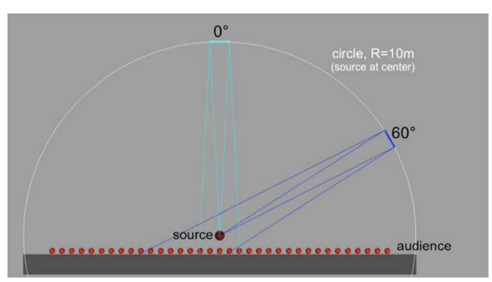

FIG. 2. (Color online) Cross section representation of a generic concert hall seating layout, displaying the effect of two hypothetical acoustic reflectors. Small dots represent audience heads. Both reflecting surfaces represent the same solid angle measured from the source, provide reflections of same loudness and delay but to differing proportions of the audience.

smaller than about 75°. With the angle definitions generally used in the context of grazing incidence attenuation, this corresponds to a grazing angle (angle with respect to the surface tangent) larger than about 15°. As a consequence, formula (9) can be considered fully valid for reflections with angles of incidence on the audience which are not too large, whereas a correction should probably be incorporated specifically for angles of incidence larger than 75°. Further research could aim at quantifying the grazing incidence attenuation as a function of the angle of incidence on the audience plane, thus taking the attenuation into account in the solid angle theory.

The solid angle theory developed here indicates that shallow incidence reflections should be favored in order to create higher average strength in large halls. However, systematically favoring shallow incidence reflections over zenithal reflections is not always a good design strategy: Firstly, for angles of incidence larger than a specific value, yet unknown but certainly superior to 75°, the attenuation of sound at mid-frequencies due to grazing incidence might become stronger than the positive effect of spreading the same acoustic intensity over a wider proportion of the audience. Secondly, grazing incidence attenuation is stronger at low frequencies. The most widely accepted criterion for the perception of bass in concert halls[22–24] is Bass Index: $\mathrm{BI} = G(125\mathrm{Hz}) - \mathrm{Gmid}$. Seat dip attenuation being especially strong near 125 Hz, providing only grazing incidence reflections would certainly impact negatively on the perception of bass.

Nevertheless, favoring shallow incidence reflections has the positive effect of providing the required amount of early reflections while leaving sufficient energy for the development of a generous late response.

IV. CONCLUSION

A new approach to early reflection design for performing arts spaces has been developed. A simple formula has been derived for the prediction of the average value of early reflected strength G_{em} across the audience in a hall.

The solid angle approach reveals implications that should be of great interest to acoustic designers. The average early reflected energy across the audience was found to depend on only three architectural parameters: the efficient solid angle (Ω_{eff}), the total area occupied by the audience (S_{aud}) and a specific average value of the angle of incidence of early reflections on audience planes (θ_m). The use of these architectural parameters by acoustic consultant in the development of innovative room shapes and new typologies of concert halls could lead to increased chances of acoustic success.

An important finding of this study is that all acoustic reflectors are not equal in efficiency. In terms of solid angle, the most efficient ones are those that are the most visible from the stage. In addition, reflective surfaces providing early reflections from a shallow angle of incidence are more effective than those providing zenithal reflections. Up to a certain point, shallow incidence reflections should then be favored in the design of performing arts spaces.

In the design of a hall, increasing the efficient solid angle up to very large values can also have some shortcomings. Beyond the risks of excessive loudness, enough energy should be left for the late reverberant field. Future developments will aim at quantifying this transfer of energy from the early to the late field, the solid angle approach being very promising for that purpose. The angle of incidence of early reflections will certainly have an influence on this energy transfer since shallow incidence reflections were found to use a smaller fraction of the sound power emitted by the sound source to provide the same average amount of early reflected energy to audience members.

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